Application of Six Sigma Management Tools to Increase the Quality of a Catapult

Project Report

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May 30, 2010
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</tbody>
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Project Report

May 30, 2010

1 Introduction
In the course *Six Sigma Quality Management*, our group develops a hand-made wooden catapult by ourselves. We try to apply six sigma management tools through define, measure, analyze, improve and control (DMAIC) phases to improve the quality of the catapult. The catapult of our project is a device used to shoot a projectile a distance with the aid of a rubber band.

2 Define
This phase aims to define the scope and goals of this six sigma project.

2.1 Define the scope
This project focuses on improving the quality of the catapult we made in the course *Six Sigma Quality Management*. The projectile of the catapult is a golf ball, which has a diameter of 4.2cm and weights 46 grams.

2.2 Define the defect
We first define what accuracy is and what precise is in our project. Accuracy is defined as the shift between the observed hitting point and the target hitting point. Precise is defined as the variation of shooting distances from the observed average. Thus, in this project, the defect is defined as the inaccurate or imprecise shooting distance of the catapult.

2.3 Define the goal
The goal of this project is to make the shooting distance of the catapult as accurate and precise as possible. Namely, the catapult can hit a specific point without bias and have acceptable variation. The target variation of the shooting distance in this project is no more than two diameters of the golf ball ($\pm 2 \times 4.2\text{cm} = \pm 8.4\text{cm}$).

3 Measure
In the measure phase, we first map the structure of the catapult. Then, we make a gage R&R analysis to assess the measurement system. Finally, process capability analysis of the baseline performance is conducted.

3.1 Mapping the structure of the catapult
Figure 1 is a picture of the catapult. Each part of the catapult is labeled in the picture. The elasticity of this catapult for casting projectile is the rubber band. The movement of the casting rod is restricted by the lower barrier rod and the upper barrier rod. To cast a ball for some distance, we can move the casting rod down to any position above the lower barrier rod. The lower barrier rod can also be removed, which provides the flexibility of the movement of the casting rod and thus gives an acceptable range of shooting distances.
The projectile container is used to contain the shot ball. Since the elasticity is high and the weight of the catapult itself is low, high weighting fixing objects are important for the stability of the catapult.

![Picture of the catapult](image)

**Figure 1 Picture of the catapult**

### 3.2 Establish measure system capability

Gage R&R analysis is conducted to assess how much variation is associated with the measurement system. During the experiment, we select 5 landing points and have 3 operators to measure these distances.

In the first try, we cast the ball to hit the ground directly. Then, operators just ‘guess’ the hitting spot by naked eyes and measure this distance by a tapeline. The results show that the total gage R&R is larger than 30% which implies that the measurement system is unacceptable. Based on this analysis, we improve the measurement system. We stick blank papers on the ground and then cover blank papers with carbon papers. Then, when the shot ball hits these carbon papers, a marker of the shot ball is formed. Figure 2 shows some markers produced by shot ball hitting.
The initial distances of the selected 5 landing points measured by 3 operators are showed in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>CL</td>
<td>197.1</td>
<td>197.2</td>
</tr>
<tr>
<td></td>
<td>LY</td>
<td>197.2</td>
<td>197.4</td>
</tr>
<tr>
<td></td>
<td>ZKM</td>
<td>197.4</td>
<td>197.5</td>
</tr>
<tr>
<td>#2</td>
<td>CL</td>
<td>198.5</td>
<td>198.5</td>
</tr>
<tr>
<td></td>
<td>LY</td>
<td>198.7</td>
<td>199.1</td>
</tr>
<tr>
<td></td>
<td>ZKM</td>
<td>198.7</td>
<td>198.9</td>
</tr>
<tr>
<td>#3</td>
<td>CL</td>
<td>196.7</td>
<td>196.5</td>
</tr>
<tr>
<td></td>
<td>LY</td>
<td>196.7</td>
<td>196.8</td>
</tr>
<tr>
<td></td>
<td>ZKM</td>
<td>196.7</td>
<td>197.1</td>
</tr>
<tr>
<td>#4</td>
<td>CL</td>
<td>197.8</td>
<td>197.6</td>
</tr>
<tr>
<td></td>
<td>LY</td>
<td>197.4</td>
<td>197.6</td>
</tr>
<tr>
<td></td>
<td>ZKM</td>
<td>197.8</td>
<td>198.0</td>
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<td>199.6</td>
<td>199.5</td>
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<tr>
<td></td>
<td>LY</td>
<td>199.7</td>
<td>199.4</td>
</tr>
<tr>
<td></td>
<td>ZKM</td>
<td>199.8</td>
<td>199.8</td>
</tr>
</tbody>
</table>

Based on this data set, we use Minitab to get more information. From Table 2 we can see that the total gage R&R of our measurement system is 16.81% which is between 10% and 30%. Since the size of the marker produced by shot ball is about 1 cm, it is not easy for operators to figure out the true center of the landing point. Thus, we decide to accept this measurement system.
Table 2 Total gage R&R of the measurement system

<table>
<thead>
<tr>
<th>来源</th>
<th>研究变异 (SD)</th>
<th>(6 * SD) 研究变异 (%SV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>合计量具 R&amp;R</td>
<td>0.19895</td>
<td>1.19373</td>
</tr>
<tr>
<td>重复性</td>
<td>0.14805</td>
<td>0.88829</td>
</tr>
<tr>
<td>再现性</td>
<td>0.13291</td>
<td>0.79745</td>
</tr>
<tr>
<td>Operator</td>
<td>0.13291</td>
<td>0.79745</td>
</tr>
<tr>
<td>部件间</td>
<td>1.16664</td>
<td>6.99986</td>
</tr>
<tr>
<td>合计变异</td>
<td>1.18349</td>
<td>7.10092</td>
</tr>
</tbody>
</table>

可区分的类别数 = 8

Moreover, the results of our measurement system analysis with the ANOVA method are showed in Table 2. Total gage R&R of the measurement system shows that the majority of the variance comes from the parts. In the R control chart, measurement of the range of the sample of all operators on all the points are within upper and lower specification limits. The measuring results of the second operator are more distributed than the other two operators. From the Xbar control chart, we can see that almost the mean values of all the measuring results are out of range, which confirms the effectiveness of our measure system. The number of distinct categories is 8, which is much larger than 4 or 5.
### Dis 的量具 R&R (方差分析)

<table>
<thead>
<tr>
<th>量具名称</th>
<th>研究日期</th>
</tr>
</thead>
<tbody>
<tr>
<td>变异分量</td>
<td>0.00</td>
</tr>
<tr>
<td>R 控制图 (按 Operator)</td>
<td>UCL=0.6350</td>
</tr>
<tr>
<td>Xbar 控制图 (按 Operator)</td>
<td>LCL=0</td>
</tr>
</tbody>
</table>

Figure 3 plots of gage R&R analysis

### 3.3 Establish baseline process capability

Process capability refers to the uniformity of the process.

The objectives of this project are to shoot as accurately and precisely as possible. The catapult is made by us. Without doing experiment, we have no idea the exact distance the catapult can hit. Thus, it’s a little bit difficult for us to pre-setup a target to hit. In this experiment, we want to estimate the variation of the distances shot by the catapult. Then we consider the mean got in experiment as the target value which means that the process center locates the target. Also we concentrate on the short term variation.

So we will mainly focus on the process capability Cp and also take other parameters as reference.

1) **Descriptive Statistics Analysis**

The descriptive statistics are listed as follows:

<table>
<thead>
<tr>
<th>变量</th>
<th>平均值</th>
<th>平均值标准误</th>
<th>标准差</th>
<th>最小值</th>
<th>下四分位数</th>
<th>中位数</th>
<th>上四分位数</th>
<th>最大值</th>
<th>极差</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dis_FirstTime</td>
<td>201.31</td>
<td>1.16</td>
<td>6.76</td>
<td>184.30</td>
<td>196.43</td>
<td>202.33</td>
<td>206.13</td>
<td>212.10</td>
<td>27.80</td>
</tr>
</tbody>
</table>

The mean is 201.31 and standard deviation is 6.76. The range is 27.80.

The box plot is as follows:
It has a more precisely performance than the first rough one we made several weeks ago.

2) Normality Test
Then we do the normality test to test whether these data we have collected are normally distributed. From the following probability plot, we can find that the p value is 0.617 which is much larger than 0.05, and the normality of these data is confirmed.

3) Detailed Analysis
Two Golf Ball Diameters Specification Limits
First we will use ± two golf ball diameters \((4.2^2 = 8.4 \text{ cm})\) as the user specification limits to estimate the process capability, and use the sample mean as the process mean:

| Table 4 Two Golf Ball Diameters Specification Limits of Dis_FirstTime |
|-------------------------|-------|-------|
| LSL                    | Mean  | USL   |
| 192.91                 | 201.31| 209.71|

Here is the process capability analysis output by Minitab:

![Dis_FirstTime 的过程能力](image)

**Figure 6 Process capability analysis of Dis_FirstTime**

From this plot, the \(Cp\) is 0.41, really small. The \(Cpk\) is the same as \(Cp\) because there is no mean shift in this system. The process still needs to be improved substantially.

4 Analyze

In this phase we will focus on analyzing causes of shooting bias.

4.1 Cause and effect analysis

Large variation has been reported in the first experiment. To reduce variations, we analyze the machine and identify some factors which may affect the variation. Finally, a fishbone diagram is formed as shown in Figure 7.
1) **Machine**

The machine is assembled manually by ourselves, and the materials we use are not fine enough to fit each other. Thus manufacturing quality of the catapult may influence the variation on the following aspects.

- Stability of the base

  Since the device is not heavy, it may shake when casting. If the catapult is not stable when casting, the variation will be enlarged.

- Fatigue strength of the rubber band

  The elasticity source of the catapult is the rubber band, which may become slack and cannot supply elastic power as before after repetitive telescoping. Thus slack rubber band may cause large variation.

- Joint of parts

  Different parts are linked by limited quantities of screws and rods, therefore the casting rod may rotate against more resistance and the holder may shake violently when casting, both of which may enlarge the variation.

2) **Casting methods**

Since the device is not made sophisticatedly, different kinds of casting methods will lead to significant differences of landing points. Some key factors might be included by the following points.

- Holding position when releasing

  When casting the ball, the operator can hold everywhere on the casting rod. But
different positions lead to different landing points. Same holding positions tend to obtain smaller variation while some other holding positions tend to obtain larger variation.

- Casting gestures

A proper gesture helps to observe the position of casting rod and make the operator feel comfortable. By this situation, the operator tends to do the job steady and accurately. On the other hand, improper casting gestures leads to larger variation.

- Direction control of the casting rod

The casting rod swings left and right because of the slack joint of axis and the holder. Poor direction control will lead to larger variation.

3) Materials

Materials of the device are not specially made. In fact, they came from our daily life. For example, we use a golf ball as the shot ball, snipped plastic bottle as the shot ball container, etc. Actually, some materials are not suitable for the device. The following three may affect the variation a lot.

- Material of shot ball

In order to get a perfect movement trace and a good landing point, the shot ball should be neither too large nor too small, and neither too heavy nor too light. Lighter balls travel longer and are easier influenced by air resistance, which leads to large variation.

- Material of casting rod

When casting, the operator pulls the casting rod to touch the lower barrier rod and then release it. If the material of casting rod is not rigid, the elastic power might be enlarged because of over pulled or reduced because of no contact, both of which lead to unstable landed points.

- Material of barrier rods

As described previously, the material of barrier rods also effect the variation. Again, if the material of barrier rod is not rigid, the elastic power might be enlarged because of over pulled or reduced because of no contact, both of which lead to unstable landed points.

4) Operator

We also found that different operators tend to obtain different variations. Operators’ operations may be influenced by different factors and we consider the following two are significant.

- Strength of operator

If the operator has more strength, he or she can hold the casting rod steadier. Thus at the moment of releasing the casting rod, the rod begins to move from the right position. On the other hand, the operator with less strength may obtain larger variation because he or she cannot hold the rod steady.
Stability of operator

Stable operator tends to obtain smaller variation while unstable operator tends to obtain larger variation.

5) Environment

Environment can affect the variation more or less in the following two aspects.

- Air resistance

The shot ball moves in the air against air resistance. Thus, if we use a lighter shot ball, such as the ping pong ball, air resistance may affect the landed point very much and will enlarge the variation.

- Floor

Hard and flat floor will help to finish the casting job steady or else the variation might be larger.

6) Measurement

No matter how accurately and steady we cast the shot ball; we have to find an effective way to measure the results. We cannot work on it with poor measurement methods.

- Mark method

After the shot ball lands, a mark should be drawn immediately so that we can measure the distance that the ball has traveled. Inaccuracy mark method will of course lead to a large variation.

- Measurement method

After the mark has been made, the observer will measure the distance by rulers, different reading methods lead to different results. Inaccuracy reading methods will lead to a larger variation.

4.2 Further analysis of causes to large variation

The objective of our catapult is to hit a specific point accurately. One key index to describe its performance is the shooting distance. As shown in Table 3, we found that the variance of the shooting distances is large, and the sample is not concentrated. As mentioned above, our objectives are to decrease variance and increase accuracy.

Thus, the key work is to find out what is concerned with variance and accuracy.

As shown in Figure 7, we know that there are many causes leading to shooting bias. Based on empirical analysis and also qualitative experimental studies, we come to the conclusion that the variation is largely dependent of the casting rod and the shooter, further, including:

- shift of casting rod perpendicular to shooting direction, which
- bend of the casting rod, which will influence shooting strength
- shooting strength, which is mainly most closely related with the shooter’s pulling strength, i.e. how much the shooter pulls the casting rod back

So the objective of our project turns to getting the casting and the shooter status more stable. The following section will focus on the details of improvement.

5 Improve

In the improve phase, we first make some tentative improvements based on results of the analyze phase and assess the process capability of our catapult. After that, an experiment is designed and the results shows that a standard and robust trigger can help a lot on reducing variation of the shooting distance. Based on these analysis and experiments, we make more improvements. Finally, we design an experiment, trying to assess the relationship of shooting distance and the distance between the trigger to the base of the catapult.

5.1 Tentative improvements

1) Casting rod

Figure 8 shows the control to the casting rod. We add sticks beside the casting rod so that it has a more stable motion curve.

2) Strength

Figure 9 shows some improvements in detail. We use a piece of rope to pull the casting rod instead of applying force on it directly; therefore the point of force applying on the casting rod is constant.

And also, we fix a point on the ground and we allocate the end of the rope to this point; thus, the strength to shoot varies much less. This job is of great help to decrease the effect of bend of the casting rod. The curve of casting rod is almost the same each time when we shoot.

Further, we made 2 types of triggers as shown in Figure 10. Especially the second one, a alteration of a tape holder, can greatly low fatigue of shooters’ finger, and make it possible to control more precisely the strength by fix the allocation of the rope end.
Figure 8 Control to the catapult lever

Figure 9 Control of shooting strength
5.2 Process capability analysis

1) Descriptive Statistics Analysis

From the descriptive statistics, we find that the mean is 213.80 and standard deviation is 4.07. The range is 15.80. The mean value is different from the one of the first time because the elastic force is enlarged after we improve the shooting part structure. What pleases us is that the standard deviation has been decreased by nearly 40%. The range has also been decreased by nearly 43%.

The box plot for the data of first and second time is:

The variance of Dis_SecondTime is obviously smaller than that of the Dis_FirstTime, which shows that our improvements are effective.
2) **Normality Test**

With normality test, we can find that the p value is 0.282 which is much larger than 0.05, and thus the normality of these data is confirmed. However, the p value is smaller than the one of the first time, and it needs to be investigated.

3) **Detailed Analysis**

**Two Golf Ball Diameter Specification Limits**

Still, we will use ± two golf ball diameters (4.2 cm) as the user specification limits to estimate the process capability, and use the sample mean as the process mean:

Table 5 Two Golf Ball Diameters Specification Limits of Dis_SecondTime

<table>
<thead>
<tr>
<th>LSL</th>
<th>Mean</th>
<th>USL</th>
</tr>
</thead>
<tbody>
<tr>
<td>205.4</td>
<td>213.8</td>
<td>222.2</td>
</tr>
</tbody>
</table>

Here is the process capability analysis output by Minitab:

![Process capability analysis of Dis_SecondTime](image)

Figure 12 Process capability analysis of Dis_SecondTime

From this plot, the Cp is 0.68, relatively larger than that of the first time. The Cpk is the same as Cp because there is no mean shift in this system. However, this Cp is also smaller than 2. After applying our tentative improvements, the variance has been decreased, but we think that further actions need to be taken to keep the stability of the shooting process with the improved device. The process still needs to be improved.

5.3 **Design of experiment**

As analyzed above, we came to the conclusion that the variation is largely dependent on the casting rod and the shooter based on empirical analysis and also qualitative
experimental studies. So here we want to find out their contributions to variation of shooting distances by performing design of experiments. Therefore, the response is the standard deviation.

In each run, we shoot 5 times. Then based on these 5 values we estimate the standard deviation of the population under this configuration and use it as a response.

There are two factors: one is largely concerned with the casting rod, called fixing types, which refers to how we fix the casting rod or its lope before shooting; the other is operator. By now we have tried 3 types to fix the casting rod:

A. Pull the end of the casting rod with the thumb and fore finger until the casting rod rightly touches the lower barrier rod.

B. Join a piece of rope on the end of the casting rod and pull by forefinger the other end of the rope to a specific position on the ground; of course we try to get the casting rod touch the lower barrier rod.

C. Using a trigger to fix the other end of the rope instead of holding on the ground.

Also, we choose three operators: Liu Ying, Zhu Keming and Fang Jiarui.

Thus, a 3-level full factorial design with 2 factors and 1 single replicate is generated as shown in Table 6. The last column is the response.

Table 6 3-level full factorial design with 2 factors

<table>
<thead>
<tr>
<th>Run</th>
<th>Order</th>
<th>PtType</th>
<th>Block</th>
<th>Shooter</th>
<th>FixingType</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>KM</td>
<td>Trigger</td>
<td>1.53</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>LY</td>
<td>Ground</td>
<td>2.31</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>LY</td>
<td>Trigger</td>
<td>1.22</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>LY</td>
<td>Hand</td>
<td>5.91</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>KM</td>
<td>Ground</td>
<td>1.85</td>
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<td>8</td>
<td>6</td>
<td>1</td>
<td>1</td>
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<td>Trigger</td>
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<tr>
<td>7</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>JR</td>
<td>Ground</td>
<td>1.89</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>1</td>
<td>1</td>
<td>JR</td>
<td>Hand</td>
<td>6.07</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>1</td>
<td>1</td>
<td>KM</td>
<td>Hand</td>
<td>5.49</td>
</tr>
</tbody>
</table>

The following table and figure show ANOVA and main effects plot given by Minitab. The contribution of Fixingtype is significant while relatively that of Shooter is not according to their p-values. And also, according to the main effects plot, we find that fixing the casting rod by hand leads to the largest STD and the performance by fixing on ground and by trigger is close. Trigger is a little better.

Table 7 ANOVA of Shooter, FixingType vs. STD

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>P</th>
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</thead>
<tbody>
<tr>
<td>Shooter</td>
<td>2</td>
<td>0.2771</td>
<td>0.2771</td>
<td>0.1385</td>
<td>1.04</td>
<td>0.432</td>
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<tr>
<td>FixingType</td>
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<td>31.9810</td>
<td>31.9810</td>
<td>15.9905</td>
<td>120.43</td>
<td>0.000</td>
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<tr>
<td>Error</td>
<td>4</td>
<td>0.5311</td>
<td>0.5311</td>
<td>0.1328</td>
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</tr>
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</table>
This experiment and its analysis indicate that operators’ performances are close to each other if they choose the same fixing type. For one operator, the key idea is to increase repeatability and it is best to use a trigger when shooting. Thus, our final improvement is to use a trigger (the tape holder) for holding the rope fixed on the casting rod.

5.4 Further process capability analysis

After DOE, we find that using a trigger for holding the rope fixed on the casting rod tends to have lower variation than the other two methods. Thus, we do another experiment to further assess the process capability of the catapult.

1) Descriptive Statistics Analysis

From the descriptive statistics, we find that the mean is 208.94 and standard deviation is 2.12. The range is 7.60. The standard deviation has been decreased by nearly 48% comparing with the second time. The range has also been decreased by nearly 52%.

The box plot for the data of three times is:
Figure 14 Box plot of Dis_FirstTime & Dis_SecondTime & Dis_ThirdTime

The variances of Dis_FirstTime and Dis_SecondTime are really larger than that of the Dis_ThirdTime, which shows that our improvements are significant.

2) Normality Test

With normality test, we can find that the p value is 0.338 which is much larger than 0.05, and thus the normality is confirmed.

3) Detailed Analysis

Two Golf Ball Diameter Specification Limits

Still, we use ± two golf ball diameters (4.2 cm) as the user specification limits to estimate the process capability, and use the sample mean as the process mean:

<table>
<thead>
<tr>
<th>LSL</th>
<th>Mean</th>
<th>USL</th>
</tr>
</thead>
<tbody>
<tr>
<td>200.54</td>
<td>208.94</td>
<td>217.34</td>
</tr>
</tbody>
</table>

Here is the process capability analysis output by Minitab:
Figure 15 Process capability analysis of Dis_ThirdTime

From this plot, the Cp is 1.31, really larger than that of the second time. The Cpk is the same as the Cp. However, the Cp is also smaller than 2. The Cp has been improved by nearly 93% than that of the second time because we have improved our trigger and fix it on the ground which leads to smaller vibration.

5.5 Assess the relationship of key X and Y

Here we’d like to check the relationship between the setting of the product and the distance we can reach.

Firstly, we assume the function being $y = f(x)$, where $y$ denote the distance we can reach, and $x$ is the distance between the trigger and the base. Since the larger the $x$, the more elastic the rubber band, we can predict than the larger the $y$. Set the range being $[30, 50]$ with the unit being cm, we take 21 settings for each 1 cm interval. For each treatment, we have 3 replications.

The sample data is showed in Table 9.

Table 9 sample data for regression

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.0</td>
<td>106.8</td>
<td>41.0</td>
<td>185.8</td>
</tr>
<tr>
<td>31.0</td>
<td>114.5</td>
<td>42.0</td>
<td>193.9</td>
</tr>
<tr>
<td>32.0</td>
<td>120.5</td>
<td>43.0</td>
<td>206.9</td>
</tr>
<tr>
<td>33.0</td>
<td>126.4</td>
<td>44.0</td>
<td>218.2</td>
</tr>
<tr>
<td>34.0</td>
<td>131.8</td>
<td>45.0</td>
<td>233.1</td>
</tr>
<tr>
<td>35.0</td>
<td>134.7</td>
<td>46.0</td>
<td>244.4</td>
</tr>
<tr>
<td>36.0</td>
<td>142.7</td>
<td>47.0</td>
<td>255.4</td>
</tr>
</tbody>
</table>
Seeing from the scatter plot of x and y of Figure 16, the relationship between x and y is positive as we predicted before, and the function seems to be square relationship. Thus in the next step we try to fit regression between y, x and x\(^2\). The model is:

\[
y = \beta_0 + \beta_1 x + \beta_2 x^2 + \epsilon, \; \epsilon \sim N(0, \sigma^2)
\]  

\[(1)\]

The result of the regression is showed in Table 10. We can see that the model fitted the sample data quite well with R-square being more than 99% and also in Table 10. The ANOVA result indicate that we may accept response variable x and x\(^2\) in the model.

### Table 10 Regression result of x and y

<table>
<thead>
<tr>
<th>x</th>
<th>Y</th>
<th>( x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.0</td>
<td>151.7</td>
<td>152.5</td>
</tr>
<tr>
<td>38.0</td>
<td>158.2</td>
<td>158.5</td>
</tr>
<tr>
<td>39.0</td>
<td>162.7</td>
<td>168.3</td>
</tr>
<tr>
<td>40.0</td>
<td>177.8</td>
<td>179.1</td>
</tr>
</tbody>
</table>

**Figure 16 Scatter plot of x and y**

The result of the regression is showed in Table 10. We can see that the model fitted the sample data quite well with R-square being more than 99% and also in Table 10. The ANOVA result indicate that we may accept response variable x and x\(^2\) in the model.

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>P Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>2</td>
<td>220904</td>
<td>110452</td>
<td>18681.37</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>60</td>
<td>355</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>62</td>
<td>221259</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Now we have the check the residuals of y. From Figure 18 we can see that the residuals are mostly normal. They seem to be randomly fitted with the value of y.
The probability plot of the residuals is showed in Figure 19. With the p-value being 0.55 larger than 0.05, we can say that the residuals are normally distributed. Thus the final regression equation is:

$$y = 263.6 - 13.92x + 0.2947x^2$$  \hspace{1cm} (2)

Reversely, we calculate the x value from the target y:

$$x = 23.6172 + \sqrt{3.3933y - 336.695}$$  \hspace{1cm} (3)

And with this setting we can get to the target. Take y of 200cm as an example, if we set the x to be 42.11 cm, we may throw the ball at the 200cm point.

Moreover, we’d like to consider the confidence interval of x instead of just one point. For n = 3, significant level at 0.05 and variance which is get from the previous sample data as 2.12 as stated in section 5.4, the confidence interval of y is [197.6, 202.4], which makes the x confidence interval being [41.89, 42.33]. With the setting of x in this confidence interval, each time we make 3 replications, the mean value of y should all be within the confidence interval and we can say we hit the target of 200 cm.

5.6 Summary of the improve phase

In this phase, through several rounds of improvements, we successfully reduce the standard deviation of the shooting distances from 6.76 to 2.12. After that, we design an experiment and obtain a rough regression model. Based on this model, we design another experiment, in order to cast the ball on a target.
1) **Experiment setting**

In this experiment, we set the target as 200cm. Then, based on the regression model we built before, the distance of the trigger to the base of the catapult is determined, which is 42.1cm.

Then, we fix the trigger in this position and repeat casting for 30 times and rule out one outlier.

2) **Process capability analysis**

After making a descriptive statistics, we find that the mean is 202.34 and standard deviation is 2.36. The range is 9.10. The mean value is different again because this time we try to hit the target 200 cm.

The box plot for the data of four times is:

![Box plot of Dis.FirstTime & Dis.SecondTime & Dis.ThirdTime & Dis.TargetKnown](image)

Figure 20 Box plot of Dis.FirstTime & Dis.SecondTime & Dis.ThirdTime & Dis.TargetKnown

The variances of Dis.FirstTime and Dis.SecondTime are really larger than those of the Dis.ThirdTime and Dis.TargetKnown, which shows that our improvement has paid back. The best one is the third one.

With normality test, we can find that the p value is 0.435 which is much larger than 0.05, and thus the normality is confirmed.

We do the target-known process capability analysis:

Table 11 Two Golf Ball Diameters Specification Limits of Dis_TargetKnown
Here is the process capability analysis output by Minitab:

![Dis_TargetKnown 的过程能力](image)

From this plot, the Cp is 1.16 too, while the Cpk is 0.84, smaller than Cp. This means there is a shift of mean value of the process, and the sample mean is 202.34, and there is a 2.34 shift of mean, which nearly equals to the standard deviation.

### 6 Control

#### 6.1 Control actions

To implement our improvements and maintain the current performance of the catapult, several control actions need to be taken.

- Heavy objects should be used to fix the base of the catapult.
- The trigger (a taper holder) should be fixed on the ground.
- Use the same shot ball.

#### 6.2 Statistical process control

SPC is the application of statistical methods to the monitoring and control of a process to ensure that it operates at its full potential to produce conforming product. Under SPC, a process behaves predictably to produce as much conforming product as possible with the least possible waste. While SPC has been applied most frequently to controlling manufacturing lines, it applies equally well to any process with a measurable output. Key tools in SPC are control charts, a focus on continuous improvement and designed experiments. In this part, we are hoping to test our system and find out whether
the landing point is being controlled.

1) Design the SPC experiment

From section 5.5, we know that the relationship of X and Y can be: $Y = 263.6 - 13.92 \times X + 0.2947 \times X^2$. Thus if we want to cast the shot ball to the landing point that is 200cm apart from the catapult, which means $Y = 200cm$, we have to release the rubber band at the point of $X = 42.1cm$.

Since our regression model is not very precise, for example, if we want to compute X when Y is known, the coefficients might be very small which brings truncation errors. Thus the X we derived is not precise and we have to try different values of X to find a proper value that produces the exact Y.

Also, as the limitation of our experiment environment, we did not take any real-time control policies when we did this experiment. This means, we could do nothing when we found that the landing point was far from the target point since we could not adjust the trigger by a short precise distance.

Actually, for this experiment, we are focusing on the past product quality to find if the past product process is been controlled, but not doing a real-time quality control.

2) Analysis of SPC result

We cast the ball for 29 times and create the control chart in Minitab as follows:

According to our definition, the distance between landing point and target point less than 2 times diameter of the shot ball can be accepted. That is, if we want to make the landing point at 200cm, the resulting landing point between 191.6cm and 208.4cm is
accepted. From the control chart we can see that all of the landing points that we measured are acceptable. But there are some situations that are not in control.

![Landing Point 单值控制图](image)

**Figure 23** Xbar control chart with points out of control

According to the criteria analysis given by Minitab, we find some uncontrolled points as follows:

Criteria 2: there are more than 9 successive points on one side of central line. The following points might be off controlled: 18, 19, 20, 21, 22, 23, and 24.

Criteria 5: there are 2 out of 3 successive points out of area B. The following points might be off controlled: 4.

Criteria 6: there are 4 out of 5 successive points out of area A. The following points might be off controlled: 5, 6, and 8.

3) **Future improvement of SPC**

From the above control chart we can see that the process is stable but not in control. In fact, because of the error of the regression model, the landing points are shifted by a short distance. As a result, we can make a real-time control policy to control the process as follows.

- Adjust the value of X by a fixed distance if the previous landing point is more than 4.2cm (one diameter of the golf ball) away from the target point.
- Find a confidence interval of X and adjust the value of X in the interval to confirm a proper value and then continue the experiment.
7 Summary

In this project, we build a wooden catapult by ourselves. Since the manufacturing quality of the catapult is not very good, we try to apply six sigma management tools through the DMAIC process to improve its quality. The quality of the catapult is defined as the accuracy and precise of the shooting distance. We first establish the measurement system capability and baseline process capability. Then, after a deep analysis, several rounds of improvements, including both intuitive improvements and design of experiments, are conducted. With these improvements, we decrease the standard deviation of shooting distances substantially and increase the process capability of the catapult significantly. Finally, some control actions are recommended for implementing this project.

8 Future work

Though we have increased the accuracy of the shooting distance and reduced its variation of the catapult dramatically, there is still a lot of space to make further improvements. We list some of them as follows:

- More precise measure systems are needed.
- Some rigid material should be used for the casting rod.
- Further experiments are needed to build a more accurate regression model of the shooting distance.
- The trigger should be improved to work more automatically.