Adaptive Charting Techniques: Literature Review and Extensions

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Summary. The continuous development of SPC is driven by challenges arising from practical applications across diverse industries. Among others, adaptive charts are becoming more and more popular due to their capability in tackling these challenges by learning unknown shifts and tracking time-varying patterns. This chapter reviews recent development of adaptive charts and classifies them into two categories: those with variable sampling parameters and those with variable design parameters. This review focuses on the latter group and compares their charting performance. As an extension to conventional multivariate charts, this work proposes a double-sided directionally variant chart. The proposed chart is capable of detecting shifts having the same or opposite directions as the reference vector and is more robust to processes with unpredictable shift directions.

1 Introduction

As an efficient tool for monitoring process status and helping identify assignable causes, Statistical Process Control (SPC) has been successfully applied in applications across diverse industries, including manufacturing, financial service (Tsung *et al.* (2007)), healthcare (Woodall (2006)), medical research and others. With more and more challenges arising from these real world scenarios, SPC techniques have been evolving continuously from the days of Shewhart (1931), when the concept of control charts was first introduced.

The challenges facing SPC are multi-faceted. Under the seemingly contrary goal of pursuing lower false alarm rates and higher sensitivity, SPC algorithms have to handle complex while practical situations like unknown shift magnitudes, high dimensionalities and process dynamics. Some control charts can be tuned to be more sensitive to specific shifts magnitudes. However, when true shift magnitudes are unknown or not constant, either assumptions have to be made regarding shift sizes or capable algorithms have to be designed to estimate such information. The curse of dimensionality is a well-known obstacle to achieving better charting performance. Process dynamics, especially when confounded with unknown parameters and high dimensionalities, have given serious challenges to the SPC

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community. In Section 2, features and current solutions to these issues will be summarized.

So far, one of the best ways to tackle the above challenging problems is to use control charts equipped with adaptive capability. Literally, adaptive control charts are smarter than ordinary charts in the sense that they can update themselves to fit for different situations. Therefore, even if initial settings of these charts are not accurate, they are expected to achieve optimality via self-adjustment. In Section 3, adaptive control charts with variable design parameters are extensively reviewed and some general guidelines for implementing these charts are provided.

In Section 4, a new multivariate adaptive control chart is proposed, which fills one of the gaps we identified in our review. Finally, Section 5 concludes this chapter with recommendations on future research topics.

2 The Challenges facing SPC

Due to the fast development of manufacturing and sensing technology, as well as the continuously increasing demand for high service levels in banking, healthcare and other industries, SPC are encountering challenges driven by versatile practical applications. In this section, we summarize several challenges along which we see active SPC research. In Section 3, we review different types of adaptive control charts, which have been proposed to tackle these challenges.

2.1 Detecting unknown or mixed-range shifts

The primary criterion for evaluating any SPC schemes is their capability in detecting process changes quickly and correctly. Given a fixed false alarm rate, the one with the shortest delay in detecting a specific shift is usually favored. While it is relatively easier to tune a control scheme to be sensitive to a specific shift, as the CUSUM chart presented in Section 2, true shifts may have unpredictable sizes and directions. Most control charts are good at detecting only shifts with particular magnitudes. For example, Shewhart-type charts, including univariate I-chart, $\overline{x}$-chart and multivariate Hotelling's T^2 chart, are more sensitive to large shifts than to small shifts; while some other schemes, such as the exponentially weighted moving average (EWMA) chart, cumulative sum (CUSUM) chart, and multivariate versions of these charts, multivariate EWMA (MEWMA) and multivariate CUSUM (MCUSUM), are more sensitive to shifts with small sizes.

The way that historical information is incorporated influences the performance of a control chart in detecting shifts of different sizes. Usually, if both latest and past observations are put together for monitoring, shifts with small sizes are more likely to be identified. The foregoing EWMA, CUSUM, MEWMA and MCUSUM are all this type of charts that accumulate historical information in certain ways to improve sensitivity. On the contrary, if merely the newest observations are studied, large shifts can be detected with a shorter delay since the information conveyed by these observations is not averaged out across a time span. The aforementioned Shewhart-type control charts possess this feature and are therefore more suitable for detecting large shifts.

More fundamentally, the settings of parameters dominate the performance of control charts over specific shift ranges (Montgomery (2005)). For example, the EWMA chart with a small smoothing parameter is more sensitive to small shifts than the same chart with a large smoothing parameter; the CUSUM chart with a certain reference value is most sensitive to that specific shift size.

Obviously, if the shift magnitude of a process is unknown or keeps varying over time, none of the above schemes, which are designed for either small or large shifts, can perform uniformly well. One possible treatment is to combine multiple charts to take care shifts in all ranges. For example, applying Shewhart control limits to EWMA or CUSUM charts to detect both large and small shifts (Lucas (1982), Albin *et al.* (1997), Chih-Min *et al.* (2000), Reynolds and Stoumbos (2005)).

Recently, adaptive EWMA charts (Capizzi and Masarotto (2003)) and adaptive CUSUM charts (Sparks (2000), Shu and Jiang (2006)) have been proposed and studied. By varying design parameters of the conventional charts, adaptive charts have been seen to be a promising tool for detecting unknown or mixed-range shifts. A more detailed review of adaptive control charts will be presented in next section.

2.2 Dealing with high dimensionalities

As processes in both manufacturing and service industries are getting more and more complex, dozens or even hundreds of correlated quality characteristics or process variables may need to be monitored simultaneously, which becomes another challenge to SPC. The false alarm rate of a chart that monitors multiple variables is much higher than when it monitors a single variable. If the false alarm rate is controlled by loosening control limits, the sensitivity of the chart will in turn be harmed. Due to the curse of dimensionality, the effect of shifts in one variable becomes less prominent when the variable is banded together with others.

Besides using the aforementioned Hotelling's T^2 chart for large shifts or the MEWMA and MCUSUM charts for small shifts, dimension reduction is a major technique to deal with high dimensional processes. Among others, principal component analysis (PCA), partial least square (PLS) and independent component analysis (ICA) are the most popular ways to extract a smaller set of variables that are representative enough for process monitoring.

PCA-based control charts try to find effective principal components (PCs), which are linear combinations of original process variables to represent the original high dimensional process. The number of PCs is usually much smaller than the number of original variables. Therefore, control charts can avoid high dimensionalities by monitoring these PCs only (Jackson (1991), Nomikos and Macgregor (1995), Mastrangelo *et al.* (1996), Runger and Alt (1996), Jolliffe (2002)). Recently, Tsung (2000) proposed to monitor dynamic processes with dynamic PCA. Instead of using fixed principal components, Tsung (2000) proposed to conduct PCA online, which is more suitable for processes with time-varying shifts. The application of dynamic PCA is also found in Yoo *et al.* (2002) and Choi *et al.* (2006).

If one is interested in monitoring a group of correlated variables, PCA might be utilized first to find representative components. While if the influence of these variables on quality characteristics is known, PLS can be performed first to identify a set of latent variables that are capable of better predicting quality variables (Macgregor *et al.* (1994)). The latent variables are linear combination of original process variables and can be monitored against process changes (Palm *et al.* (1997)). Nomikos and Macgregor (1995) introduced PLS-based methods that integrate initial setup condition, process variables and product variables together for process monitoring. A discussion on choosing latent variables in PLS was given by Li *et al.* (2002).

Ding *et al.* (2006) presented an example in which ICA outperforms PCA during data reduction. Different from PCA that tries to maximize explainable variation, ICA searches for components that can cluster data into distinct groups. Lee *et al.* (2003) integrated ICA with MEWMA for process monitoring; Albazzaz and Wang (2004) applied ICA-based SPC techniques to batch operations. Satisfactory results have been demonstrated by the authors.

It is becoming gradually common for profiles to be collected to characterize product quality or process status in many processes. Profiles reflect functional relationships between a response variable and one or more explanatory variables. Compared with a multivariate dataset, profiles contain even more data point and have to be modeled as an extremely high dimensional problem. Both model-based and model-free methods have been proposed for profile monitoring (see Woodall *et al.* (2004) for an extensive survey).

As a mathematical tool in signal processing, wavelet transformation has been applied to filter and decompose profile-type variables or quality characteristics (Ganesan *et al.* (2004), Jeong *et al.* (2006)). Wavelet transformation has the potential to reconstruct signals on multiple scales. Therefore, SPC schemes can be developed to monitor wavelet coefficients of different resolutions to detect slow or fast changes in underlying processes (Bakshi (1998), Ding *et al.* (2006)). Even though different wavelet basis functions can be chosen, such functions do not take engineering knowledge of original signals into consideration. Instead, model-based methods have been developed to model profile data for SPC. For example, Williams *et al.* (2007) advocated fitting nonlinear models to profiles first and then monitoring fitted parameters.

Recently, Wang and Tsung (2005) introduced an application where the quality of a surface is to be monitored. The authors borrowed profile-monitoring techniques to monitor product surface images. However, much information has been lost when modeling a surface as a profile. New technologies are to be developed for similar applications.

In Section 3, we review the most recent research on adaptive multivariate charts, which is more effective in monitoring high-dimensional processes.

2.3 Monitoring processes with dynamic means

A usual assumption behind most SPC schemes is that the normal status of the target process is stable. Means of manufacturing processes are usually assumed to be constant at a specific level when no assignable causes exist. However, there are situations under which a "normal" operational status changes over time. For example, the via etch process studied by Spitzlsperger *et al.* (2005) has drifting mean due to aging effects and its mean resets every time chamber maintenance is conducted. Such natural drifting trends must be decoupled from sampled signals so that true process faults can be identified.

To monitoring processes with dynamic means, adaptive control charts have been successfully applied in the literature. Section 3 gives examples of adaptive charts for monitoring such processes.

2.4 Detecting dynamic shifts

Dynamic shifts could be generated due to process autocorrelation, inertia properties or the existence of feedback controller (Tucker *et al.* (1993), Wang and Tsung (2007b)). Compared to sustained shifts, shifts that change over time are more difficult to capture. One way to detect time-varying shifts is to calculate smoothed trends while ignore local details. Atienza *et al.* (2002) proposed to use a backward CUSUM chart to monitoring autocorrelated observations. However, if the dynamic shifts happen to oscillate around the in-control process means, the smoothed signals lose useful information for shift detection.

Some researchers appeal to model-based methods (Lin and Adams (1996), Montgomery and Mastrangelo (1991), Pandit and Wu (1993), Apley and Tsung (2002)). By fitting time series models first, process shifts can be estimated by one-step-ahead predictions. Control charts based on residuals have been thoroughly investigated. For example, Lu and Reynolds (1999) applied EWMA charts to monitor residuals obtained from a model-based forecasting. However, the performance of model-based methods depends heavily on the accuracy of the time series models. Poor estimates of model parameters inevitably lead to poor forecasting accuracy.

Most of the above research focuses on detecting dynamic shifts in univariate processes. Apley and Tsung (2002), Tsung and Apley (2002), Wang and Tsung (2007a) and Wang and Tsung (2007b) studied the detection of dynamic shifts in multivariate processes. Since the performance of some multivariate charts is dominated by not only shift magnitudes but also shift directions, we will investigate this issue extensively in following sections and propose a new double-sided directionally variant chart for multivariate process monitoring.

Table 1 summarizes the above discussion and classifies the literature based on their primary purposes.

Table 1. Recent development of SPC along different directions

Objective	New Techniques and Literature
Detecting unknown or mixed-range shifts	Univariate charts: adaptive EWMA (Capizzi and Masarotto (2003)), adaptive CUSUM (Sparks (2000))
	Multivariate charts: adaptive T^2 chart (Wang and Tsung (2007a), Wang and Tsung (2007b))
Dealing with high dimensionalities	Dynamic PCA (Tsung (2000), Yoo et al. (2002), Choi et al. (2006)) ICA (Ding et al. (2006)) Wavelet-based methods (Ganesan et al. (2004), Jeong et al. (2006)); Profile-targeted methods Linear profile models (Kang and Albin (2000), Kim et al. (2003)) Nonlinear profile models (Williams et al. (2007)) Surface (Wang and Tsung (2005))
Monitoring processes with dynamic means	Batch process monitoring (Spitzlsperger et al. (2005))
Detecting dynamic shifts from a stable process	Cuscore (Shu et al. (2002)); RF-Cuscore (Han and Tsung (2006))
	Dynamic T^2 (Tsung and Apley (2002)); Adaptive T^2 (Wang and Tsung (2007a), Wang and Tsung (2007b))

3 Adaptive Control Charts for Process Monitoring

In control chart design and implementation, there are two sets of parameters need to be determined. The first set includes the sample size and the sampling interval, which can be classified as sampling parameters since settings of these parameters directly influence the way a control chart is operated; the second set contains design parameters of charting statistics, for example, the reference parameter that a CUSUM control chart is designed for, the intended shift magnitude that an EWMA control is designed for.

Adaptive control charts, in the same fashion, can be classified into two categories: those with adaptive sampling parameters and those with adaptive design parameters. Several review works on adaptive control charts is seen in the literature. Tagaras (1998) reviewed the development of adaptive charts until 1998. However, the author surveyed mainly univariate charts. In addition, only sampling parameters, including the sample size, the sampling interval, are allowed to be variable (the control limits are allowed to change with these parameters as well). Another broad review presented by Woodall and Montgomery (1999) covered some methods with variable sampling schemes. Zimmer *et al.* (2000) compared the performance of Shewhart-type control charts with adaptive sample sizes and/or sample intervals.

In practice, using control charts with adaptive sampling parameters needs the cooperation of operators, since their ordinary working pace or procedures might be interrupted or altered. In contrast, adaptive design parameters can be easily realized as long as SPC schemes are implemented with the aid of computers. Automated software can be designed to adapt to continuously updated parameter without affecting practical operational procedures. In this review, we focus on the second category of adaptive charts, that is, control charts with adaptive design parameters.

Adapting design parameters of control charts is driven by practical situations in which true process parameters are critical to charting performance while impossible to obtain. In the following, we review adaptive control charts and group them based on the way that critical design parameters are estimated and updated.

3.1 Recursive estimation of in-control mean/variance/covariance parameters

Some researchers have classified the control chart implementation procedure into two distinct stages: Phase I and Phase II (Woodall (2000)). In Phase I, historical data are collected and parameters are estimated. Control charts are then set up based on estimated parameters. Phase II involves online monitoring and signaling (Woodall (2000), Wang and Tsung (2005)).

However, as noted by Spitzlsperger *et al.* (2005), obtaining data in Phase I to reach acceptable level is too costly for some applications. While if insufficient number of samples is used in Phase I, large uncertainties may be seen in parameter estimation. Jones (2002) studied the design of EWMA charts with estimated parameters; Jones *et al.* (2004) evaluated the run length performance of CUSUM charts when parameters are estimated. Jensen *et al.* (2006) reviewed the impact of parameter estimation on control chart properties. Aside from all the work that tries to quantify the impact of parameter estimation uncertainties on ARL performance, a seemingly better alternative is to improve estimation accuracy by using samples collected in Phase II. This idea has been adopted by some adaptive control charts.

Spitzlsperger *et al.* (2005) applied Hotelling's T^2 chart to monitor a via etch process on a dual-frequency capacitive coupled parallel plate machine. Due to the aging effects and periodic chamber maintenance, some variables show slow drifts and quick jumps periodically.

Let $\mathbf{z}_i$ be a vector of the i th sample, $i = 1,...,n$. Each element of $\mathbf{z}_i$ is standardized with respect to sample mean μ_j and standard deviation σ_j . However, as μ_j and σ_j are unknown in practice, the authors estimated and updated μ_j via

$$\hat{\mu}_j = \lambda x_{ij} + (1-\lambda)\hat{\mu}_{j-1} ,$$

and updated σ_j via

$$\hat{\sigma}_j^2 = \frac{p-2}{p-1}\hat{\sigma}_{j-1}^2 + \frac{1}{p}(x_{ij} - \hat{\mu}_j)^2 ,$$

where λ is a smoothing parameter. By recursive estimation of such parameters, the resulting chart is capable of compensating aging effects and detecting true process faults.

3.2 Recursive estimation of out-of-control mean shifts

Unlike process parameters that can be estimated in Phase I, true process shifts are only available when the process is already running and assignable causes have occurred. As some control charts can be designed to be most sensitive to a specific shift, online estimation of process shifts becomes crucial to those charts (Capizzi and Masarotto (2003), Sparks (2000)).

As noted by Sparks (2000), the CUSUM statistic can only be optimized if accurate information on a specific sustained process shift is known. The derivations in Section 2 also demonstrate the way that CUSUM relies on a specific reference parameter, δ . Therefore, under situations where such information is not available, algorithms must be designed to predict the future value of δ and tune the chart for predicted values.

Sparks (2000) proposed an adaptive CUSUM chart, which monitors the following statistics:

$$z_t = \max\left[0, z_{t-1} + \left(x_t - \delta_t / 2\right)/h(\delta_t)\right].$$

Compared with the conventional CUSUM chart, the adaptive CUSUM procedure replaces the constant term, δ , by time-varying statistics, δ_t. A new function $h(\delta_t)$ is added to maintain a constant control limit. The shift magnitude, δ_t, is online updated via an EWMA-type equation,

$$\delta_t = \max(wx_{t-1} + (1-w)\delta_{t-1}, \delta_{\min}) .$$

The author suggested taking $\delta_{\min} = 0.5$ for detecting smaller shifts and $\delta_{\min} = 1.0$ for detecting shifts larger than 1.0. The ARL performance of the adaptive CUSUM chart is studied by Shu and Jiang (2006).

As the smoothing parameter of a traditional EWMA control chart is usually chosen according to intended shift magnitudes, Capizzi and Masarotto (2003) proposed an adaptive EWMA procedure and suggested using a variable smoothing parameter, which is determined based on estimated shift magnitudes. In specific, the adaptive EWMA takes the following form,

$$z_t = (1 - w(e_t))z_{t-1} + w(e_t)x_t ,$$

where $e_t = x_t - z_t$, and $w(e_t)$ is a function of e_t. For small values of e_t, $w(e_t)$ takes a relative small value; while when e_t is large, the value of $w(e_t)$ becomes large accordingly. Therefore, the adaptive EWMA can adjust its smoothing parameters according to estimated shift sizes.

3.3 Recursive estimation of shift directions of multivariate processes

Shifts in multivariate processes are characterized by not only shift magnitudes but also shift directions. Some multivariate control charts can therefore be designed to be sensitive to shifts along a specific direction. If the ARL performance of a chart depends on the mean vector and variance-covariance matrix only through the non-centrality parameter, the chart is directionally invariant. In contrast, if the performance is also influenced by shift direction, the chart is directionally variant (Lowry and Montgomery (1995), Wang and Tsung (2007a)). MEWMA, MCUSUM and Hotelling's T^2 charts all belong to the latter category.

Hawkins (1993), Jiang (2004b) and Zhou *et al.* (2005) considered the following format for a directionally variant chart:

$$T_d^2 = \mathbf{d}^T \mathbf{\Sigma}^{-1} \mathbf{x}_t > h, \tag{1}$$

where $\mathbf{d}$ is a constant vector that indicates the direction along which the control chart is optimized. Jiang (2004a) proposed a U_0 chart and a U_∞ chart for monitoring feedback-controlled processes. The U_0 and U_∞ charts can be proved to be directionally variant charts designed for shifts identified for representing transient and steady-state status. However, in order to implement the above charts, specific and constant shift directions must be identified.

Zhou *et al.* (2005) proposed to combine both directionally variant and invariant charts together for process monitoring. The directionally variant charts are designed for most likely directions along which shifts may occur; the directional invariant chart is in place to take care of general shifts:

$$\begin{cases} T_1^2 = \mathbf{d}_1^T \mathbf{\Sigma}^{-1} \mathbf{x}_t > h_1 \\ T_2^2 = \mathbf{d}_2^T \mathbf{\Sigma}^{-1} \mathbf{x}_t > h_2 \\ T^2 = \mathbf{x}_t^T \mathbf{\Sigma}^{-1} \mathbf{x}_t > h_3 \end{cases}$$

If any one of the charts signals, the process is diagnosed as out-of-control. Similar to the chart in Equation (1), Zhou *et al.* (2005)'s method requires some specific shifts to be known in advance. Furthermore, the simultaneously functioning multiple charts have overlapped regions. For example, a large shift along direction $\mathbf{d}_1$ is expected to trigger both T_1^2 and T^2 charts to signal. Such an overlapped design can obviously harm the overall sensitivity of the scheme.

Recently, Wang and Tsung (2007b) and Wang and Tsung (2007a) proposed an adaptive T^2 chart:

$$T^2 = \mathbf{d}_t^T \mathbf{\Sigma}^{-1} \mathbf{x}_t - \frac{1}{2} \mathbf{d}_t^T \mathbf{\Sigma}^{-1} \mathbf{d}_t > h, \tag{2}$$

where $\mathbf{d}_t$ is a time-varying vector that indicates the direction along which the control chart is optimized for.

As $\mathbf{d}_t$ is unknown in practice, Wang and Tsung (2007a) proposed two ways of forecasting $\mathbf{d}_t$. The first one is model-based forecasting. An ARMA(1,1) time-series model was fit to the data sequence and the one-step-ahead prediction is obtained, which is used to estimate $\mathbf{d}_t$. The second one is model-free EWMA smoothing. The authors update $\mathbf{d}_t$ recursively by

$$\mathbf{d}_t = \lambda \mathbf{x}_t + (1-\lambda)\mathbf{d}_{t-1}.$$

Based on simulation results, the authors found that EWMA-based forecasting is more robust to model misspecification.

To tackle strong oscillations found in feedback-controlled processes, Wang and Tsung (2007b) proposed an oscillated EWMA for shift forecasting. Similar to the EWMA forecasting with uses exponentially decaying weights, the oscillated EWMA uses decaying weights with alternative signs. That is,

$$\mathbf{d}_t = \lambda \mathbf{x}_t - (1-\lambda)\mathbf{d}_{t-1}.$$

When applied to a data sequence with signals go up and down alternatively, the oscillated EWMA can pickup such trend and enhance useful signals for fault detection.

Table 2 summarizes the adaptive charts that have variable design parameters.

Table 2 Adaptive charts with variable design parameters

Adaptive parameters	Control charts and representative literature
Shift magnitude	Adaptive EWMA (Capizzi and Masarotto (2003)) Adaptive CUSUM (Sparks (2000)) GLRT (Apley and Shi (1999))
Process mean/covariance matrix	Adaptive PCA (Tsung (2000)) Hotelling's T^2 (Spitzlsperger et al. (2005))
Shift pattern and shift direction	Dynamic T^2 (Tsung and Apley (2002)) Adaptive T^2 (Wang and Tsung (2007a), Wang and Tsung (2007b))

One fundamental assumption behind the above study is that the process is undergoing a sustained mean shift. Therefore, directionally variant charts show good performance in designated areas and the adaptive T^2 chart with EWMA forecasting exhibit satisfactory results in detecting small shifts. However, if process shifts are difficult to be estimated accurately, the performance of the above charts will be deteriorated. In the following section, we propose a double-sided directionally variant chart that takes the advantageous of both directionally variant charts and adaptive charts. The charting performance of the new chart is expected to be superior to other charts when double-sided shifts are to be detected.

4 A double-sided directionally variant chart for multivariate processes

It has been noted in previous sections that the multiple-chart scheme proposed by Zhou *et al.* (2005) has overlapped detection regions and failed to fully utilize the capability of all control charts. The adaptive T^2 chart due to Wang and Tsung (2007a) is good at detecting shifts that can be forecasted by EWMA equations. However, at any step, the adaptive chart is still a one-sided chart. If true process mean shifts are difficult to be estimated via EWMA, such as the feedback-controlled process discussed by Wang and Tsung (2007b), the resulting performance may be seriously deteriorated.

To improve the performance of the adaptive T^2 chart, there are two possible solutions: one way is to find intelligent forecasting algorithms so that the shifts can be estimated more accurately; the other way is to design robust charts to reduce the adverse effect of estimation uncertainties. Accurate forecasting algorithms may be case-dependent. For example, the oscillated EWMA proposed by Wang and Tsung (2007b) is suitable for strongly oscillated signals. Designing a robust control chart, on the other hand, may be a generally fitted solution.

Aiming to compensate the low efficiency of one-sided directionally variant chart, we propose a double-sided directionally variant chart. This chart takes care of both positive and negative shifts. This chart preserves the flexibility of the adaptive T^2 chart in estimating process shifts while uses doubled-sided statistics to avoid missing unexpected shifts in opposite directions.

We start from the directionally variant chart in Equation (1). Let $\underset{\sim}{\mathbf{d}}$ be an estimated shift direction and is standardized, $\underset{\sim}{\mathbf{d}}^T \mathbf{\Sigma}^{-1} \underset{\sim}{\mathbf{d}} = 1$, let $\underset{\sim}{\mathbf{d}}^{-}$ be the opposite of $\underset{\sim}{\mathbf{d}}$, $\underset{\sim}{\mathbf{d}} = -\underset{\sim}{\mathbf{d}}^{-}$. We define two charting statistics,

$$T_1^2 = \underset{\sim}{\mathbf{d}}^T \mathbf{\Sigma}^{-1} \mathbf{x}_t$$

and

$$T_2^2 = (\underset{\sim}{\mathbf{d}}^{-})^T \mathbf{\Sigma}^{-1} \mathbf{x}_t$$

We know that the chart T_1^2 is capable of detecting all shifts that satisfy

$$\boldsymbol{\mu}^T \mathbf{\Sigma}^{-1} \underset{\sim}{\mathbf{d}} > 0 \tag{3}$$

while T_2^2 is capable of detecting all shifts that satisfy

$$\boldsymbol{\mu}^T \mathbf{\Sigma}^{-1} \underset{\sim}{\mathbf{d}}^{-} > 0 \tag{4}$$

Since $\underset{\sim}{\mathbf{d}} = -\underset{\sim}{\mathbf{d}}^{-}$ holds, the region characterized by Equations (3) and (4) covers all possible shifts.

We now define the double-sided directionally variant chart as

$$T_D^2 = \max(\mathbf{d}_{\sim}^T \mathbf{\Sigma}^{-1}\mathbf{x}_t, (\mathbf{d}_{\sim}^{-})^T \mathbf{\Sigma}^{-1}\mathbf{x}_t) > h$$

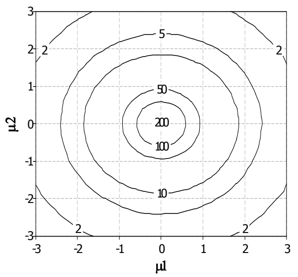
As T_1^2 and T_2^2 follow the same statistical properties and $\mathbf{d}_{\sim} = -\mathbf{d}_{\sim}^{-}$, the above equation can be further simplified as

$$T_D^2 = |\, \mathbf{d}_{\sim}^T \mathbf{\Sigma}^{-1}\mathbf{x}_t \,| > h$$

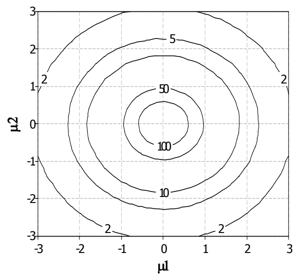
Analogous to the adaptive T^2 chart, we estimate the shift via EWMA updating

$$\mathbf{d}_t = (1-w)\mathbf{d}_{t-1} + w\mathbf{x}_t \, .$$

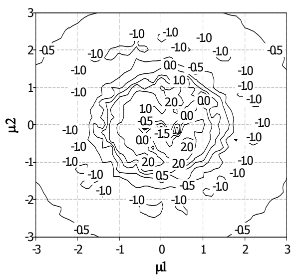
Once $\mathbf{d}_t$ is obtained, $\mathbf{d}_{\sim}$ will be derived as



(a) Adaptive T^2



(b) Double-sided directionally variant chart



(c) ARL difference between the double-sided chart and the adaptive T^2 chart

Figure 1 Performance comparison between the double-sided directionally variant chart and the adaptive T^2 chart

$$\mathbf{d}_{\sim} = \frac{\mathbf{d}_t}{\sqrt{\mathbf{d}_t^T \mathbf{\Sigma}^{-1} \mathbf{d}_t}},$$

which is standardized to satisfy $\mathbf{d}_{\sim}^T \mathbf{\Sigma}^{-1} \mathbf{d}_{\sim} = 1$ at each step.

We now conduct simulations to investigate the performance of the newly proposed double-sided chart and compare it with the adaptive T^2 charts. A bivariate process with an identity covariance matrix is studied. Different from the above studies, dynamic shifts rather than sustained and constant shifts are applied to the process. At each step, shift vectors, with a sustained shift magnitude, are flipped with a probability of 50% to simulate shifts that EWMA cannot forecast accurately. The ARL contour plots of the adaptive T^2 chart are shown in Figure 1 (a), which is seen to expand away from the center and implies that the sensitivity of this chart has decreased. Figure 1 (b) shows the ARL performance of the double-sided directionally variant chart under the dynamic shifts described above. The ARL contour plots show that even though the proposed chart employs directionally variant statistics, the chart itself is invariant to shift directions. Compared with the adaptive T^2 chart, as Figure 1 (c) shows, the proposed chart is advantageous in detecting moderate and large shifts.

4 Conclusions

This chapter has summarized the recent challenges that are facing SPC, including unknown or mixed-range shifts, high dimensionalities, and process dynamics. To better tackle these challenges, adaptive control charts with variable sampling and design parameters have been reviewed. In specific, we have emphasized those charts with adaptive design parameters.

Adaptive control charts are ideal for meeting the above challenges due to its fundamental capabilities: learning and tracking. The learning capability of a control chart makes it possible for unknown shifts to be estimated. As a result, control charts can be optimized to detect such shifts. The tracking capability makes it possible for dynamic shifts to be captured.

This chapter has also proposed a double-sided directionally variant T^2 chart. The newly proposed chart possesses the flexibility of an adaptive T^2 chart and is more robust to shift oscillated seriously and cannot be accurately forecasted via EWMA.

It is learned from our review that research efforts are still needed to design better learning and tracking algorithms. Current EWMA-based or time-series-model-based methods are not sufficient for general purposes. Such issues are important topics for future research.

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